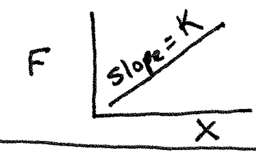
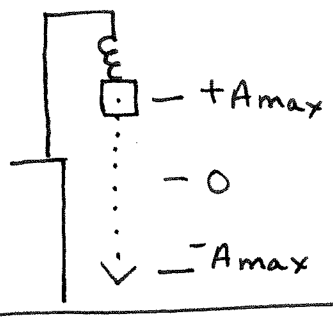
AP Physics Review

SHM

Cool Stuff:
ω = angular speed = sec^{-1}



Scenario #1: A mass on a spring



Big HINTS:
- Mass affects Period of This Pendulum.
- ω is in radians so set your calculator for radians Mode !!!

What We Know:

E_{Total} is constant as spring-mass oscillate

E_T: KE + U at any given instant. $\boxed{E_T \text{ depends on Amplitude}}$

KE_{max} = when mass is @ $y = 0$ (Max Velocity too)

U_{max} = when mass is @ $y = +A$ or $-A$ (zero velocity)

$$K_{spring} = \frac{mg}{x} = \frac{4\pi^2 mass}{T^2}$$

$$T = 2\pi\sqrt{\frac{m}{K}} \quad \text{(T has units of seconds)} = \frac{2\pi A}{V_{max}} \text{ or } sec^{-1}$$

$$\omega = 2\pi f = \sqrt{\frac{K}{m}} \quad (\omega \text{ has units of Radians/sec})$$

$$f = \frac{1}{T} \quad \text{(f has units of Hz or } sec^{-1})$$

What we can find out:

1. Find spring constant K

$$K = \frac{mg}{x} = \frac{4\pi^2 m}{T^2}$$

hang a mass and measure period

hang a mass + measure displacement

2. Find position y at any instant (Again, set calculator for <u>RAD</u> Mode!)

$$y = A\cos\omega t = A\cos\left(\frac{2\pi t}{T}\right) = A\cos(2\pi f t) = A\cos\theta \leftarrow \text{starts @ } \pm A = y_0$$

$$y = A\sin\omega t = A\sin\left(\frac{2\pi t}{T}\right) = A\sin(2\pi f t) = A\sin\theta \leftarrow \text{starts @ } y_0 = 0$$

t = time elapsed

T = period

$\boxed{\text{RAD MODE}}$

3. Find Velocity at any instant

$$V_{inst} = \pm\sqrt{\frac{K}{m}(A^2 - x^2)} = \omega A\cos\left(\omega t + \frac{\pi}{2}\right) \leftarrow \text{Starts @ } y_0 = \pm A$$

$$= 2\pi f\left(\sqrt{A^2 - x^2}\right) = \frac{2\pi}{T}\left(\sqrt{A^2 - x^2}\right) = \omega\sqrt{A^2 - x^2}$$

$$V_{max} = \pm\sqrt{\frac{K}{m}}(A) = \omega A \leftarrow \left(\text{recall } \omega = \sqrt{\frac{K}{m}}\right) = 2\pi f A$$

4. Acceleration @ any instant

$$a = -\omega^2 A\sin\omega t = -\omega^2 x = \frac{-K}{m}A\sin\omega t$$

5. Energy at any instant.

$$E = KE + U = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

As x approaches A, Velocity drops to zero

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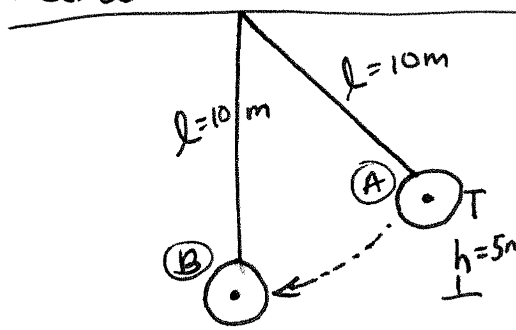
Scenario #2 Mass on a string



What we Know:

- E_{Total} is constant as string swings
- $E = KE + U$
- KE_{max} at bottom of swing
- U_{max} @ Top of either side.
- $h=$ vertical distance from position A to Position B

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$f = \frac{1}{T}$$

What we can find out:

Velocity max: $KE_{max} = U_{max}$

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$$

$$V = \sqrt{2gh}$$

$\cos\theta = \frac{h}{l}$

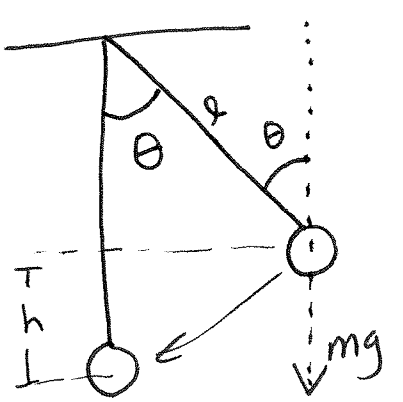
$r = l$ (length of rope)

Tension in string:

$$\text{Tension} = mg\cos\theta + F_c = mg\left(\frac{h}{l}\right) + \frac{mv^2}{r}$$

$$\text{Tension} = \frac{mgh}{l} + \frac{m(2gh)}{l} = \boxed{\frac{3mgh}{l}}$$

Wow!



Just for Fun: What is Tension @ bottom of swing?

$$\text{Tension} = mg\cos\theta + \frac{mv^2}{l} \quad (\theta = 0°)$$

$$\text{Tension} = mg + \frac{mv_{max}^2}{l}$$

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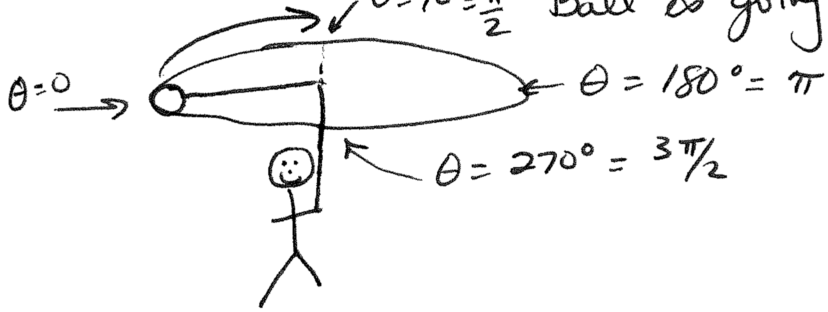
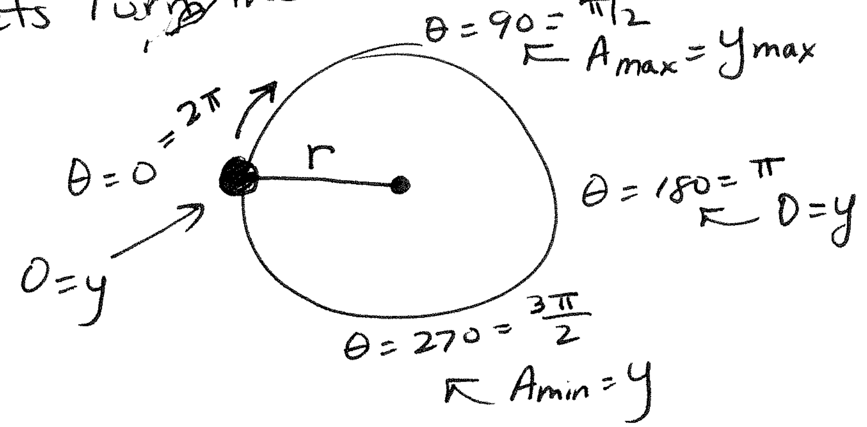
Scenario # 3: Computing SHM of a ball on a string

Ball is going around + around



Let's Turn the circle on it's side for easier viewing:



$\boxed{\text{radius} = A_{max}}$

$y = A \sin \omega t$

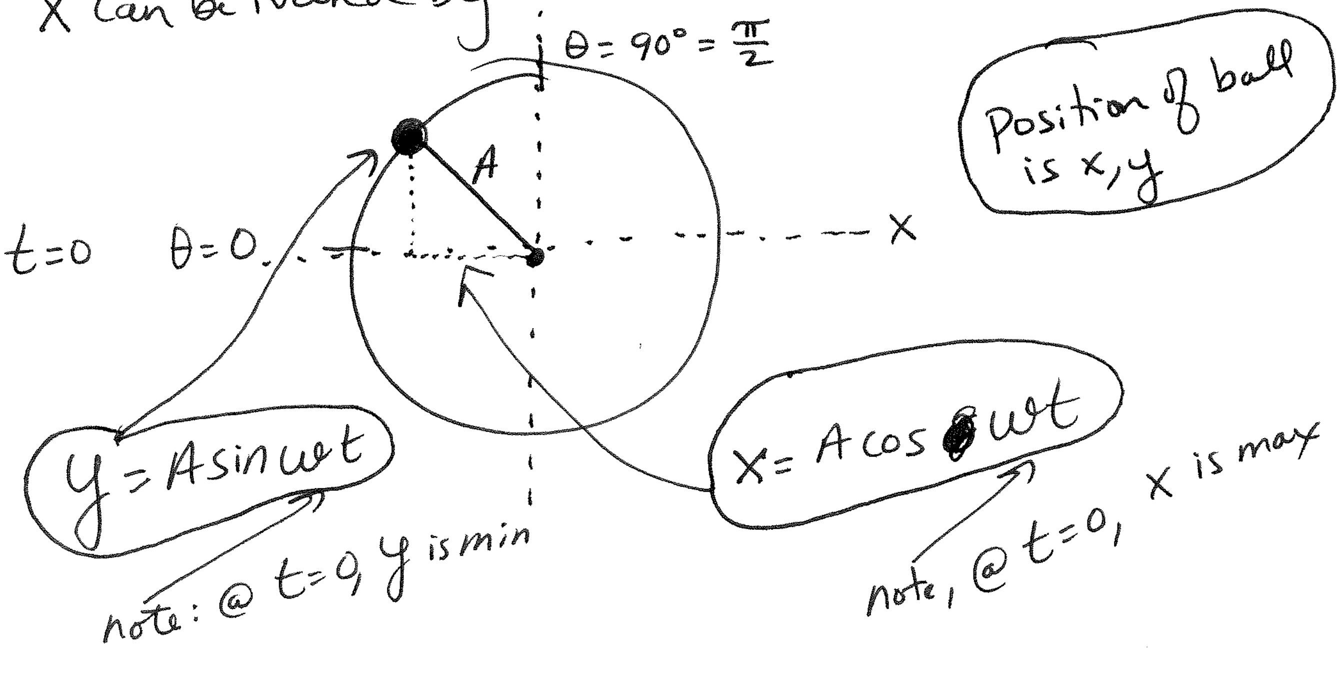
$= r \sin \omega t$

$= r \sin \theta t$ ← RAD if θ is in radians

Sin because we start @ $y = 0$

- ω = Time it takes to go around circle once $(\sec^{-1})$
  $= 2\pi f = 2\pi \left(\frac{\#\text{ of revolutions}}{1 \sec} \right)$
- Circumfrence $= 2\pi r$

X can be found by: $X = A \cos \omega t = r \cos \omega t$



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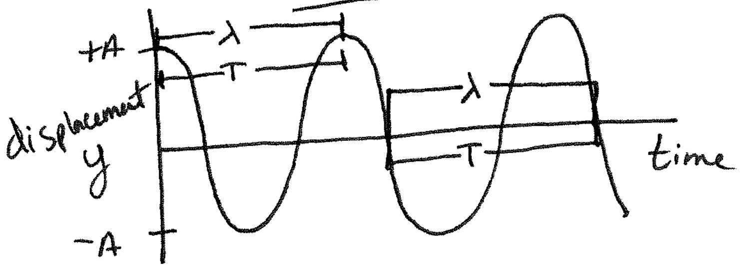
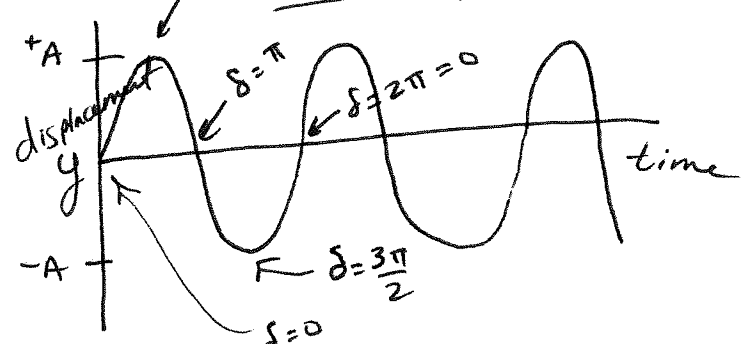
Reading Sin + Cos graphs

COS Graph



SIN Graph



λ = wavelength = distance between 2 crests or 2 troughs or 2 nodes in same direction.

T = Period = Time it takes for 1λ to pass by

f = frequency = number of λ's per second

V = wave velocity = λf = $(\text{meters})\left(\frac{1}{\text{sec}}\right)$

COS Graph

$y = A\cos\omega t = A\sin\omega t + \frac{\pi}{2}$

RAD MODE

$V_{inst} = \omega A\cos(\omega t + \frac{\pi}{2})$ ← Why $\pi/2$? because Velocity is 90° out of phase w/ displacement

V_{inst} = Tangent to The line @ any point

T = Time between Wavelengths

$f = 1/T$

SIN Graph

$y = A\sin\omega t = A\cos\omega t + \frac{\pi}{2}$

RAD MODE

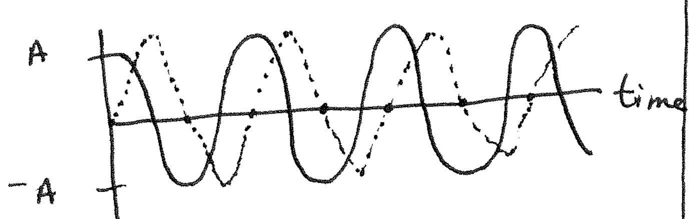
$V_{inst} = \omega A \sin(\omega t + \frac{\pi}{2})$

V_{inst} = Tangent to line @ any point.

$\boxed{\text{REmember, you can replace } \omega \text{ with } 2\pi f \text{ or } \frac{2\pi}{T}}$

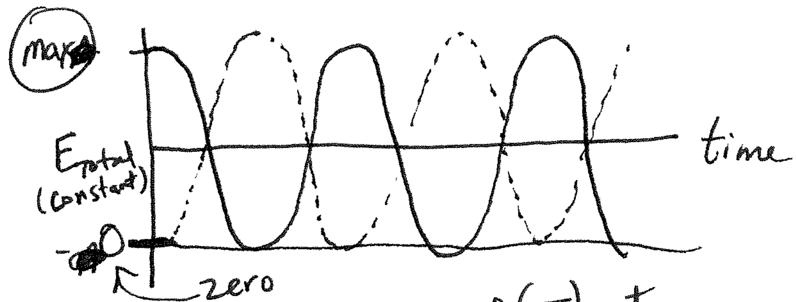
Compare graphs

Displacement — vs. Velocity - - - -



Velocity is 90° ($\frac{\pi}{2}$) out of Phase w/ displacement. Think Mass on a spring

KE — v. Potential E . . . .



KE and potential are 180° (π) out of phase w/ Each other. E_{total} is The sum of KE + U @ any instant